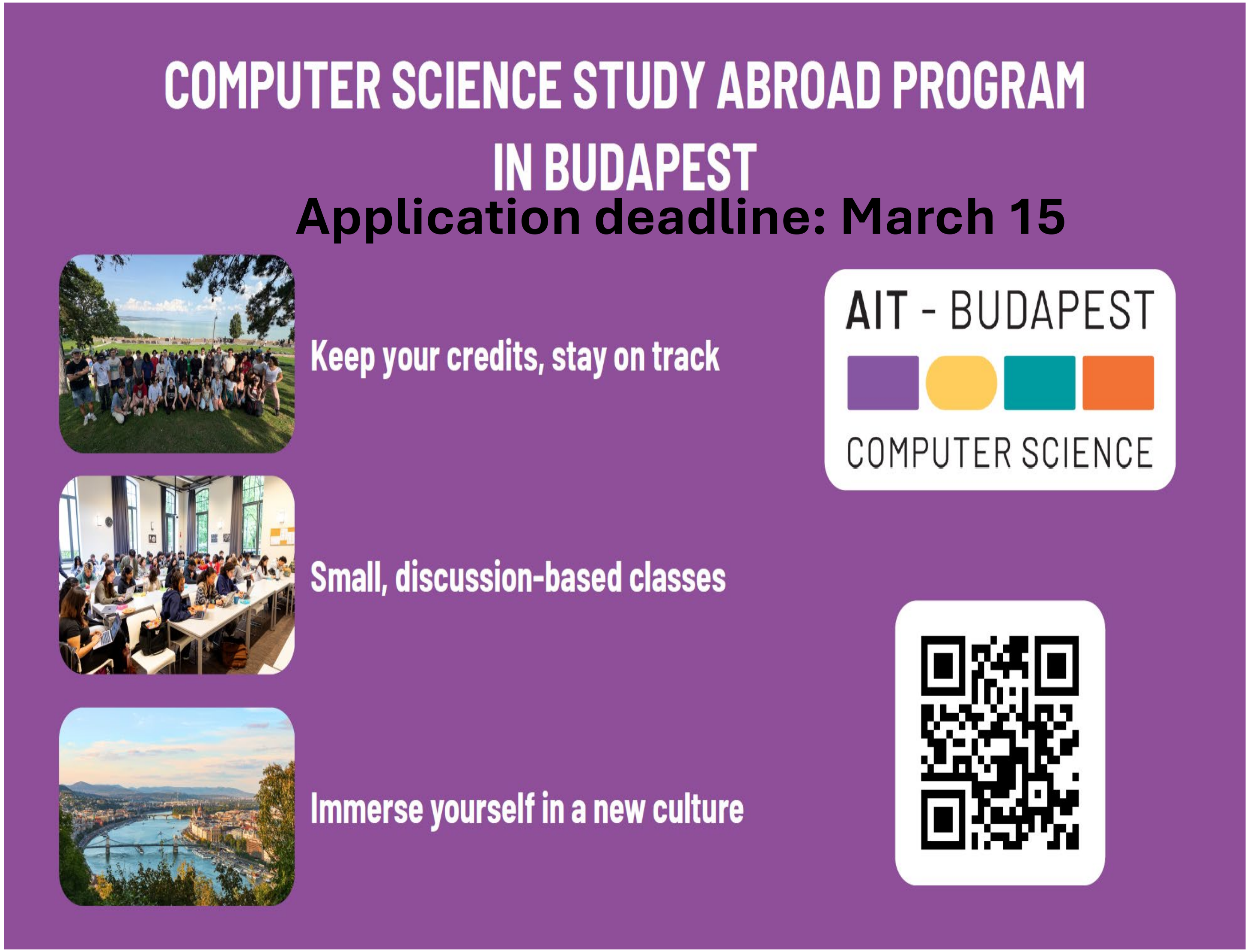
Announcements

- Hw5 interview question changed!!!
- Mid-term course eval and separate: survey on sections (email from me with the link)

  54 responses so far
- Study abroad program opportunity focusing in CS



Independent set, vertex cover and $X \leq_P Y$

two problem X & Y $\quad X \leq_p Y$ if a poly time algorithm for Y allow us to solve X in polynomial time

project selection $\leq_p$ max flow

max matching $\leq_p$ Independent set

input undirected $G=(V,E)$

$I \subseteq V$ indep if no edges between two nodes in I

find max size indep. set

input matching

G

$\Rightarrow$

input indep set equivalent

G'

$\hat{e}$

M in G matching if & only if M in G' indep. set

P and NP

P = yes/no decision problem solvable in polynomial time

e.g. • Does G have a perfect . matching

• given G & k : does G have matching of size k

• does flow network have an s→t flow of value ≥ v

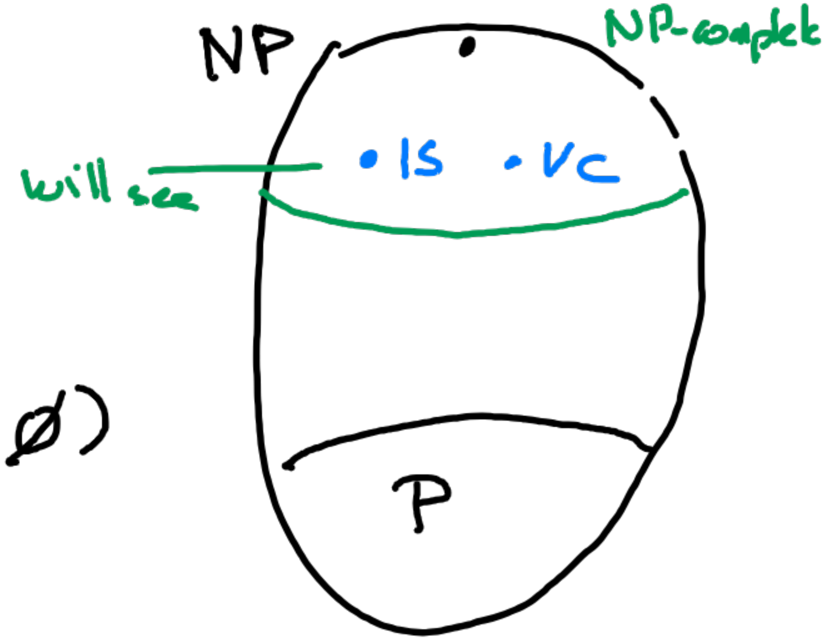
input

NP = yes/no decision problems, where if answer yes than with extra input "help" & one can verify the answer in polynomial time.

e.g. Indep set, decision version: input G = (V, E) & k

given hint $I \subseteq V$ indep of size k, easy to check

P, NP and NP-complete



Claim: $P \subseteq NP$

Proof: X problem in P, no hint (hint $= \emptyset$)

NP-complete = hardest problem in NP

problem X is NP-complete if $X \in NP$ & $\forall Y \in NP$ $Y \le_p X$

Claim: if X is NP-complete & X solvable in polynomial time $\implies P = NP$

Proof: any Y in NP, X is NP-complete, so $Y \le_p X$

$\underset{\text{def } \le_p}{\implies}$ Y solvable in polynomial time

Open $1M

Millennium Technology Prize

From “Computers and Intractability”
by Garey and Johnson

“I can't find an efficient algorithm, but neither can all these famous people.”

A first NP-complete problem SAT and 3-SAT

SAT = satisfiability $\quad x_1 \dots x_n$ Boolean varian variables

x_i = true or false

$$\underbrace{\phi}_{\text{formule}} = (x_1 \vee x_2) \wedge (x_1 \vee \bar{x}_2) \wedge \underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)} \wedge (x_2 \vee x_3)$$

formule
set of clauses
connected by AND

literal x_i or $\bar{x}_i$
$\bar{x}_i$ = opposite x_i
$\bar{x}_i = \neg x_i$

clause : set literals connected by OR

variables

Input: n variables, m clauses using these variables
Is there a way to set the variables T or F to make formule true

Eg. $\quad x_1 = T \quad x_2 = T \quad x_3 = F$

Claim SAT $\in$ NP

3-SAT special case all clauses have 3 literals

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Are the following formulas satisfiable?

$$\Phi = (x \vee y) \wedge (\bar{x} \vee z) \wedge (\bar{y} \vee \bar{z})$$

x=T
z=T
y=F

$$\Phi = (x \vee y) \wedge (\bar{x} \vee z) \wedge (\bar{y} \vee z) \wedge (\bar{z})$$

x=T   y=F   z=F

- A: Both satisfiable
- B: One is satisfiable the other one is not
- C: neither is satisfiable

A first NP-complete problem SAT and 3-SAT

Theorem (Levin & Cook 1971)
SAT is NP-complete ← will prove it end of course

Given NP-complete problem.
Claim X is NP-complete & $Y \in NP$ & $X \le_p Y$
$\Rightarrow$ Y is NP-complete

Proof. consider $Z \in NP$ need to prove $Z \le_p Y$
what we know: $Z \le_p X$ $\quad X \le_p Y$ $\quad \overset{?}{\Longrightarrow} Z \le_p Y$
X NP-complete given

Proof: Solving Z in poly time doable if we had alg for X
X solvable in poly time if alg for Y given

Independent Set is NP-complete

decision version

Proof:

Step 1: Indep Set $\in$ NP ✓

2. Pick a problem know to be NP-complete

SAT

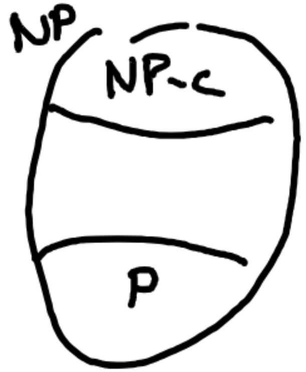
3. Show: SAT $\leq_p$ Indep Set

(will show Indep Set in NP-complete)

Summary so far

defined $P \subseteq NP$

NP-complete



Recepie to prove things NP-complete

Plans: went thru list of many different NP-complete problems

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To prove SAT $\leq_P$ Independent Set, which of the following should we do

need to show solving SAT using indep set algorithm

- A: take input to SAT $x_1, \ldots, x_n$ and the clauses and create an equivalent input to Independent set
- B: take input graph $G = (V, E)$ and k for Independent set and create an equivalent input to the SAT problem

proves IS $\leq_P$ SAT, not know